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CONSUMER EXPENDITURE FUNCTIONS FOR NON-DURABLE
GOODS: AN ECONOMETRIC STUDY

by



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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled CONSUMER EXPENDITURE FUNCTIONS FOR NON-DURABLE GOODS: AN ECONOMETRIC STUDY submitted by Venkata Bhaskara Narasimha Sastry Madduri in partial fulfilment of the requirements for the degree of Master of Arts.



ABSTRACT

Two types of demand studies, namely, demand for individual commodities by the single equation approach and the allocation aspect of consumer demand with complete system of demand equations are analyzed in this thesis. These demand equations are analyzed in terms of expenditure equations using the contemporary Canadian data for the period 1956 to 1965 covering nineteen non-durable goods. Variable coefficient models are used to capture the dynamic aspects of the demand functions.

This thesis strongly supports the specifications of proportional habit formation coefficients in dynamic demand functions. The linear expenditure system also yielded reasonable results.

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CHAPTER I

INTRODUCTION

The main purpose of this study is to explore Canadian consumer behavior with three different model specifications. The Canadian consumer is represented in a traditional model utilizing the theoretical properties of demand analysis. In the second place the same consumer behavior is studied under the influence of changes in tastes and habits over time. Finally, focusing only on non-durables, the linear expenditure system is analyzed in relation to the Canadian data.

The first model is traditional and is also called the fixed coefficient model. This is in conformity with Houthakker's [11,12] double logarithmic form except a linear form is assumed here. The consumer expenditure on any particular commodity group is a linear function of total income in real terms (real disposable income) and the relative price index of that commodity. The parameters are fixed, that is the elasticity of demand with respect to price and income are constants over time. This traditional model fails to allow for changes in tastes and habits that can influence consumption patterns. A residual time trend variable is introduced as an exogenous variable to take care of these residual variations. Thus the quantity demanded is not only a function of income and relative price

but also a function of an exogenous variable that changes over time.

The second model is the traditional model with variable coefficients which can be considered as a dynamic form of demand functions. So far the Canadian data has not been exposed to this model. The variable coefficients are in two forms. First, the parameters are linear functions of time [28]. The demand elasticities are no longer constants but represent the systematic changes of tastes and habits of the consumers. The second one is the habit formation model [17]. Most economists agree that past consumption habits will influence present consumption expenditure [22]. In this case a change in prices or incomes will cause a change in consumption which will induce a change in tastes, which will cause a further change in consumption. The parameters are functions of consumption expenditure in the previous period. A special case of this habit formation model is the proportional habit model, in which parameters are proportional to consumption in the previous period. This is expected to represent the changes in tastes or habits or the introduction of new commodities in the market better than any other formulation tried before. The demand equations are also analyzed by introducing proportional habit coefficients along with the residual trend as an explanatory variable.

The third model is the linear expenditure system of demand equations on which much empirical work has been done

by Stone [22,23,24,29]. So far there has been no attempt on linear expenditure systems of demand functions utilizing Canadian data. Only a fixed coefficient model is attempted here with expenditure restricted to non-durables. The utility function underlying this system of demand functions is additive implying a kind of independence among the commodity groups [17]. The expenditure on any non-durable commodity is a linear function of total expenditure on non-durable goods and all prices. The expenditures on any commodity is the sum of two expenditures; the expenditure on the necessary quantity of that good at current prices, and a proportion of the supernumerary income measured by total expenditure on all non-durable goods less total committed expenditure.

Data were obtained from Mr. A.S. Forti, Dominion Bureau of Statistics, Ottawa. The data cover the quarterly expenditure seasonally unadjusted, on non-durable goods, at constant dollars, divided into 19 categories. The price indexes are available for all the categories at 1957 base year. The quarterly data on the population of Canada, the real disposable income (personal income less personal direct taxes in real terms) and the total expenditure on all non-durable goods at the constant dollar values are obtained. The data are transformed into per capita figures and analyzed both in aggregate and per capita terms for comparison. Our main forms are aggregate expenditure functions and for the purpose of this study these 19 categories of non-durable

goods are grouped into four main classes. The ordinary least squares method of estimation is adopted for analyzing the fixed and variable coefficient models treating them as single equation multiple regression models. Seasonal quarterly dummy variables are introduced to represent temporal effects between different quarters. The linear expenditure system makes use of two stage iterative procedure with a basic ordinary least squares method.

The second chapter deals with the theoretical part of the three models and the functional form that is used in this study. The results and meaning of the elasticities of the fixed coefficient model and variable coefficient model are explained in Chapter III. Chapter IV reports the results of several iterations on a linear expenditure system. The final chapter is for conclusions that are derived from this study. The grouping of the commodities, the aggregated group expenditures and the aggregate group price indices are set out in the Appendix I.

CHAPTER II

THE MODELS

The specification of the relationships to be studied is the subject matter of this chapter. Specification of the demand models depend on the type of empirical investigation. Demand studies can be classified into two major groups:¹ "To the first group belong those studies which concentrate on an empirically acceptable explanation of demand for individual commodities, while the overall relationships between the quantities demanded of all commodities in the budget remain in the background. The second group of studies is chiefly concerned with the allocation aspect of consumer demand and has complete systems of demand equations as its object. The overall restrictions on demand equations provided by the theory of consumers' choice play a dominant role in these studies."²

The first kind of demand studies are concerned with finding a model where the quantity demanded (or expenditure on that commodity) of an individual commodity is explained by its own price, price of its related commodities, and total income. This is the approach followed by many

¹ An exposition of classical demand theory is available in [9] pp. 6-34.

² Barten [5], pp. 213.

investigators, who worked with Canadian data, like Holmes [10], Houthakker [11], and Reddy [21]. Two of the models considered here, namely, fixed coefficient model and variable coefficient model, belong to the first group of demand studies. The linear form of Houthakker's double logarithm system of demand functions is followed because of its simplicity of calculations.¹

To compare both types of demand studies mentioned earlier, we have also estimated linear expenditure systems. Under the assumption of the existence of individual demand functions, we shall assume that market demand functions are theoretically plausible. The analysis is based on such aggregate demand functions. "Any system of demand functions implies a corresponding system of expenditure functions [17]". This thesis will focus on expenditure functions. The three models that are investigated are elucidated below.

Fixed Coefficient Model or Static Demand (or Expenditure) Functions

A number of investigators have worked on Canadian consumer demand functions with fixed coefficients using time series data. H. Schultz [30], A. Powell [19], R.A. Holmes [10], H.S. Houthakker [11] and others are pioneers

¹ Even though this system does not satisfy the theoretical properties of demand analysis, "this function remains without serious rivals in respect to goodness of fit, ease of estimation, and immediacy of interpretation" Houthakker [11], p. 278.

in obtaining demand elasticities of selected commodities.

Powell used a simple linear model with expenditure on a particular commodity group as the dependent variable and the total expenditure and prices of its own and other commodities as explanatory variables. Holmes explored demand for beef and pork and used per capita data to adjust for population changes. He introduced trend as an explanatory variable to take care of changes in tastes and preferences. A relatively simpler form is followed by Houthakker. The dependent variable is per capita expenditure on goods and services at constant dollars, the explanatory variables are relative price and total expenditure on all goods and services per head at the constant dollars. Ordinary least squares method of estimation is used to estimate the parameters. The Bank of Canada [2] fitted a regression equation on consumer expenditure on all non-durable goods. The fitted regression is highly aggregated for all non-durables.

A linear functional form is a first approximation to express quantity demanded as a function of the explanatory variables. The linear form is assumed because of ease of estimation. The theory specifies that the quantity demanded is effected not only by its own price but also by the prices of all other related goods. The principal explanatory variable used in this analysis to capture the effect of variation in other prices is the relative price index. Even theoretically there are strong arguments for

using this variable rather than all the prices [11]. The introduction of relative price index is not uncommon.¹ In the present study the relative price index is defined as the price index of that commodity group divided by the price index of all non-durable goods. The other explanatory variable is real disposable income, which is equal to personal income minus personal direct taxes deflated by the consumer price index of all goods and services at 1957 dollars. The dependent variable is not the quantity demanded but the expenditure in constant dollar terms on that particular commodity group. Thus the first formulation of the fixed coefficient model at time t for the i th commodity group is,

$$E_{it} = e^i(Y_t, P_{it}) \quad (1)$$

where, E_i = Expenditure on i th commodity group at constant dollars.

Y = Real disposable income.

P_i = Relative price index of the i th commodity.

In explicit linear form,

¹ See for example Holmes [10] and Reddy [21]. In Holmes study the relative price index is defined as the ratio of pork price index to the consumer price index. Reddy used the relative price index as the principal explanatory variable in determining the elasticities of demand of non-durable goods.

$$E_{it} = a_i + b_i Y_t + c_i P_{it}$$

Hereafter this is called a linear expenditure equation. Let b stand for the income coefficient and c for price coefficient in all specifications.

The distribution of expenditure on any commodity group is governed not only by the real disposable income and relative prices but also by the changes in tastes and preferences, habits, and new commodities in the market etc. There are certain dynamic factors [28] which include a resistance on the part of the consumer to immediate changes in his habits which may be demanded by changes in his circumstances, the effects of stocks of commodities held by the consumer, and the fact that consumers take into account not only changes that have occurred but also those that they expect to occur. An important change in the structure arises from the direct and indirect effects on tastes and habits at any time. Changes in tastes and habits are expected to change the expenditure slowly and so they can be represented to a first approximation by introducing time as an explanatory variable in the expenditure equation. Thus a linear expenditure equation should be representable of the variations in income, changes in prices, changes in tastes, preferences and habits of the consumer¹ along with his family composition. Such an expenditure equation forms

¹ The dynamic theory is further explored by Pollak [18].

the second formulation in this study. It can be written as,

$$E_{it} = e^i(Y_t, P_{it}, T) \quad (2)$$

T is the residual trend, $T=1,2,3,\dots,n$.

The complex of preferences of families varies not only because of variations in the inherent tastes of individuals of the family but also because of the variations in the age and sex composition of the family. Treating the data in per capita terms is expected to reduce this variation reasonably. Thus the first two models are evaluated with both aggregate and per capita data.

Variable Coefficient Model or Dynamic Model

Stone [28] advocates that changes in tastes and habits are extremely important over time in determining the amount demanded of any commodity. According to him variables like changes in family size are less important than the changes in tastes. It has been noticed in U.K., during 1928-38 the number of smokers increased by 32% while the adult population increased by only 9%. The increase in smokers is due to the changes in habits which offset the increase in population. Changes in tastes and habits may be reflected in the response to economic variables, income and prices. Farrell [5] noticed that over time "the response to a given rise in income may be different to the

response of a corresponding fall, and that the same holds true for responses to changes in prices. Thus if the income elasticity is positive but greater for a rise than for a fall in real income, then the mere variation of real income over a given range will lead to an increase in consumption at all levels of income. If a taste is being developed for a particular commodity, it may be hard to return to the previous level of consumption when real income falls back to its previous level while the full opportunity may be taken of an increase in real income to increase consumption still further. It seems highly plausible to suppose that some kind of changes in tastes and habits act via income and prices in this way".¹ There should be some method to represent these effects in the expenditure (or demand) equation. Stone [28] suggests the dynamic demand functions. The main fact that emerges from this discussion is the contrast between the influence of income and prices in static situation and the way in which changes in tastes and habits effect consumer behavior.

The simplest way to represent this behavior is to make the parameters (the coefficients of income and the coefficients of relative prices) time dependent. Thus the dynamic nature of the coefficients will reflect the changes in tastes and habits on income and prices which govern the consumer behavior. The income and price elasticities of

¹ Stone [28], p. 271.

expenditure are no longer constant over time but indicate the systematic changes in tastes and habits. This can be done in two ways.

The parameters depend on any variable which take pre-assigned values at time t . One such variable is time itself. The simplest possible is to introduce T linearly. Thus the coefficient b_i of income and the coefficient c_i of relative price for the i th commodity can be written as,

$$\left. \begin{aligned} b_{it} &= b_i^* + b_i^{**} T \\ c_{it} &= c_i^* + c_i^{**} T \end{aligned} \right\} \quad (3)$$

This is called the linear trend model [23,24].

This change is incorporated in the fixed coefficient model (equation 1) set up earlier. "Thus the new model can reflect either an increasing taste for, or improved possibilities of, say, transportation, though it cannot reflect a taste for anything which first rises and then falls".¹

An attempt is also made to compute the systematic changes on one of the parameters independently, if any, keeping the other fixed.

$$\text{i.e.,} \quad b_{it} = b_i^* + b_i^{**} T; \quad c_{it} = c_i^* \quad (4)$$

¹ Stone [28], p. 51.

and,
$$b_{it} = b_i^*; \quad c_{it} = c_i^* + c_i^{**} T \quad (5)$$

The second type of variable coefficient model is to make c and b functions of some appropriate combination of past expenditure [22,28]. It has been argued that past consumption patterns are important in determining the present consumption patterns. The linear trend model implies that the changing tastes will continue to increase the expenditure in spite of prices and incomes remained constant over a long period of time. Pollak and Wales [17] set up a habit formation model where each parameter is a linear function of the past consumption expenditure. The expenditure on any commodity can be separated as physiologically necessary and psychologically necessary expenditure [18]. The physiologically necessary expenditure is the minimum expenditure every consumer has to spend to satisfy his physical needs. The psychologically necessary expenditure is due to changes in tastes, habits, new commodities in the market that influence the consumer's psychology. The coefficients of a habit formation model can be written as,

$$\left. \begin{aligned} b_{it} &= b_i^* + b_i^{**} E_{it-1} \\ c_{it} &= c_i^* + c_i^{**} E_{it-1} \end{aligned} \right\} \quad (6)$$

E_{it-1} is the expenditure on the i th commodity in period

($t-1$). Here b_i^* is the physiologically necessary and $b_i^{**} E_{it-1}$ the psychologically necessary components of b_{it} . A positive value for b_i^{**} or c_i^{**} indicates an increasing expenditure due to the changes in tastes, habits, and preferences etc., that are reflected through changes in income and prices.

Goods may be habit forming so that the consumers' present preferences depend on his past consumption habits. In this case a change in income or price will cause a change in consumption which will induce a change in tastes, which will further change consumption and so on [18]. A special case of this habit formation model is the proportional habit model where the constant term in equation (4) is suppressed. The consumer is completely represented by his psychological needs only. This reflects the changes in tastes and habits directly by past consumption pattern. According to Pollak "the necessary quantity of each good is proportional to that good in the previous period".¹ That is,

$$\left. \begin{aligned} b_{it} &= b_i^{**} E_{it-1} \\ c_{it} &= c_i^{**} E_{it-1} \end{aligned} \right\} \quad (7)$$

and

The coefficients b_i^{**} and c_i^{**} are called "habit formation

¹ Ibid., p. 749.

coefficients" which is that proportion of the past consumption expenditure carried out in time period t .

In general the habit formation model can be written as,¹

$$b_{it} = b_i^* + b_i^{**} Z_{it-1}; \quad c_{it} = c_i^* + c_i^{**} Z_{it-1} \quad (8)$$

where Z_{it-1} is a variable representing consumption of the i th good prior to period t . Z_{it-1} is either dependent on past consumption or on the rate of growth of consumption. In a more general way Z_{it-1} can be expressed as dependent on a geometrically weighted average of all past consumption of that good. In such cases,

$$Z_{it} = (1-\delta) \sum_{j=0}^{\infty} \delta^j E_{it-1j} \quad 0 < \delta < 1$$

where δ is the memory coefficient.

In this study the linear trend model, habit formation model and the proportional habit models are analyzed.

¹ Details are available in Ibid., p. 750.

Linear Expenditure System

The methods discussed earlier do not meet all the conditions laid out by theory. A more sophisticated model which satisfies all these properties of demand analysis is the linear expenditure system which was first estimated by Stone [22]. Stone pioneered in the empirical evaluation of this linear expenditure system based on U.K. data [23, 24,29]. Later Powell [19], Yoshihara [32], Pollak and Wales [17], and Pollak [18] developed the system under different conditions. The system of demand equations proposed and estimated by Stone are as follows. The quantity demanded of the i th commodity X_i is,

$$X_i = c_i + \frac{b_i}{P_i} \left(Y - \sum_{j=1}^m P_j c_j \right), \quad i=1,2,\dots,m \quad (9)$$

The corresponding expenditure equation is obtained by multiplying the above quantity demanded by its own price,

$$E_i = P_i X_i = P_i c_i + b_i \left(Y - \sum_{j=1}^m P_j c_j \right) \quad (10)$$

These demand functions correspond to the Klein-Rubin [14] constant utility index of cost of living. A simple interpretation can be given for the above system. The equation states that the expenditure on the i th commodity is equal to a certain basic consumption c_i valued at the existing prices plus a certain proportion b_i of total expenditure

less total committed expenditure called the supernumerary income [32]. "The consumer first uses up a certain amount of total expenditure in acquiring the consumption vector $c = (c_1, c_2, \dots, c_m)$ at current prices, and then distributes the remainder over the set of available commodities in a certain fixed proportion given by the elements of $b = (b_1, b_2, \dots, b_m)$ ".¹ If the b 's are all positive and the income is greater than the committed expenditure $(Y - \sum_j P_j c_j)$, it means that the consumer is purchasing the necessary quantities of the goods and then dividing his supernumerary income $(Y - \sum_j P_j c_j)$ among the goods in fixed proportion. Here the b_i 's and c_i 's are the parameters of the system and b_i is the derivative of expenditure on the i th commodity with respect to income. The parameter c_i is interpreted as the subsistence level or permanent level of consumption for the i th good, hence $P_i c_i$ represents the expenditure for this basic level of consumption. The second term $b_i(Y - \sum_j P_j c_j)$ represent the amount of supernumerary income spent on the i th good. If there are no committed quantities of expenditure then the vector c is a null vector. In such cases the linear expenditure system reduces to a simple model,

$$E_i = P_i X_i = b_i Y \quad (11)$$

¹ Ibid., p. 263.

Stone refers to equation (11) as the naive model and the model described by equation (9) or (10) as the Sophisticated model.

The demand equations are homogeneous of degree zero in prices and income; they satisfy the adding up criterion; and the Slutsky symmetry condition. The second order conditions will be satisfied if $(Y - \sum_j P_j c_j) > 0$ and $0 < b_j < 1$ holds for all j . These properties of linear expenditure system are illustrated below.

Under the assumption that positive quantities of commodities are bought, the proportion of expenditure spent on each commodity out of income in the model yields all b_i 's positive. This rules out inferior goods in the system. Consider the utility function [14].

$$U(X) = \prod_{k=1}^m (X_k - c_k)^{b_k}$$

$$\text{or, } \log U(X) = \sum_{k=1}^m b_k \log (X_k - c_k) \quad b_k > 0 \quad (X_k - c_k) > 0$$

Maximizing this utility function subject to the budget constraint $Y = \sum_i P_i X_i$, we obtain the demand function of the form,

$$\begin{aligned} X_i &= c_i - \frac{b_i}{P_i} \sum_k P_k c_k + \frac{b_i}{P_i} Y \\ &= c_i + \frac{b_i}{P_i} (Y - \sum_k P_k c_k) \end{aligned}$$

The corresponding expenditure of the i th commodity is,

$$E_i = P_i X_i = P_i c_i + b_i (Y - \sum_k P_k c_k)$$

which is the same as equation (10). Summing over all m commodities in the naive model we get,

$$\sum_i P_i X_i = \sum_i b_i Y$$

Hence,

$$\sum_{i=1}^m b_i = 1$$

Thus the parameter b_i is restricted by the condition $0 < b_i < 1$ and $\sum_i b_i = 1$. These two conditions together with the budget constraint satisfy the additivity condition. The homogeneity condition can be satisfied by multiplying P_1, P_2, \dots, P_m and Y in equation (10) by a constant, the factor of proportionality,

$$KP_i X_i = KP_i c_i + b_i (KY - \sum_j KP_j c_j) \quad K \neq 0$$

or

$$P_i X_i = P_i c_i + b_i (Y - \sum_j P_j c_j)$$

Hence the equilibrium condition is unaltered. To check

the Slutsky condition¹ we carry out the appropriate differentiation to obtain:

$$\frac{\partial X_i}{\partial P_j} = - \frac{b_i b_j}{P_i} ; \quad \frac{\partial X_j}{\partial P_i} = - \frac{b_j c_i}{P_j}$$

and

$$\frac{\partial X_i}{\partial Y} = \frac{b_i}{P_i} ; \quad \frac{\partial X_j}{\partial Y} = \frac{b_j}{P_j}$$

The symmetry condition requires that,

$$\frac{\partial X_i}{\partial Y} X_j + \frac{\partial X_i}{\partial P_j} = \frac{\partial X_j}{\partial Y} X_i + \frac{\partial X_j}{\partial P_i}$$

$$\frac{\partial X_i}{\partial Y} X_j + \frac{\partial X_i}{\partial P_j} = \frac{b_i}{P_i} [c_i + b_j (\frac{Y}{P_j} - \frac{1}{P_j} \sum P_k c_k)] - \frac{b_i c_j}{P_i}$$

$$= \frac{b_i c_j}{P_i} + \frac{b_i b_j}{P_i P_j} (Y - \sum_k P_k c_k) - \frac{b_i c_j}{P_i}$$

$$= \frac{b_i b_j}{P_i P_j} (Y - \sum_k P_k c_k)$$

¹ This section comes from Yoshihara [32].

$$\begin{aligned}
\frac{\partial X_j}{\partial Y} X_i + \frac{\partial X_j}{\partial P_i} &= \frac{b_j}{P_j} [c_i + b_i \left(\frac{Y}{P_i} - \frac{1}{P_i} \sum_k P_k c_k \right)] - \frac{b_j c_i}{P_j} \\
&= \frac{b_j c_j}{P_j} + \frac{b_i b_j}{P_i P_j} (Y - \sum_k P_k c_k) - \frac{b_j c_i}{P_j} \\
&= \frac{b_i b_j}{P_i P_j} (Y - \sum_k P_k c_k)
\end{aligned}$$

Hence,

$$\frac{\partial X_i}{\partial Y} X_j + \frac{\partial X_i}{\partial P_j} = \frac{\partial X_j}{\partial Y} X_i + \frac{\partial X_j}{\partial P_i}$$

The income elasticity of expenditure for the i th commodity group is given by

$$\eta_i = \frac{\partial X_i}{\partial Y} \cdot \frac{Y}{P_i X_i} = \frac{b_i Y}{P_i X_i}$$

Similarly, the income elasticity of expenditure for the j th commodity group is given by

$$\eta_j = \frac{\partial X_j}{\partial Y} \cdot \frac{Y}{P_j X_j} = \frac{b_j Y}{P_j X_j}$$

The elasticity of substitution between the commodities i

and j is,¹

$$\eta_{ij} = \frac{(b_i - \delta_{ij})b_j Y}{P_j X_j P_i X_i} (Y - \sum_k P_k c_k)$$

where $\delta_{ij} = 1$ if $i=j$
 $= 0$ if $i \neq j$

Since b_i and b_j are positive η_{ij} is always positive for a positive supernumerary income, and η_{ii} is negative if $0 < b_i < 1$. This satisfies the second order conditions of Slutsky matrix. Thus the linear expenditure system passes all the theoretical tests. This can be classified under the "Complete System of Theoretically Plausible Demand Functions".²

The disadvantage of the linear expenditure system is that it operates only under certain restrictive conditions. The system can only be described in a world of substitutes from which complementary and inferior classes of commodities are excluded [22]. Further the method of estimation followed by Stone is a two stage iterative procedure by applying ordinary least squares method of estimation. The estimated parameters are not reliable in the sense that no meaningful standard errors are computable by this method. Parks [16] suggested an efficient method of estimation which

¹ See Stone [29], p. 204.

² Pollak and Wales [17], p. 611.

is a modified method of Stone's procedure by deleting one of the redundant equation and applying the generalized least squares method. He claims that by this modified method when convergence is reached they are the maximum likelihood estimates. Pollak and Wales [17] estimated the parameters of the linear expenditure system by maximum likelihood method of estimation. Both the investigators incorporated variable coefficients in the model.

CHAPTER III

EMPIRICAL RESULTS FOR THE FIXED AND VARIABLE COEFFICIENT MODEL

The data used for the estimation of the models stated earlier are quarterly time series on Prices, Expenditure and Disposable income for the period 1956-65 based on 1957 dollar prices and not seasonally adjusted. Final consumption expenditure on nineteen non-durable goods of the Canadian economy are considered. These nineteen categories of non-durable goods are grouped into four classes of commodity groups for ease of calculation. The commodity groups are: Food, Clothing, Household goods, and Pleasure goods. The method of grouping is explained in Appendix I.

Seasonal dummy variables S_1, S_2, S_3 are introduced in the model so that the estimated coefficients are free from seasonal variation in the data. The ordinary least squares method of estimation is used with the normal assumptions regarding the disturbances.¹

In general, changes in population size are expected to change consumption expenditures. As the number of members in a family increases more food, more clothing, etc., are required. To incorporate the changes in population which are due to the changes in family size and composition

¹ Details are given in Johnston [13], Ch. 4.

Asimakopulus [1] introduced family size as an explanatory variable. Holmes [10], Houthakker [11,12] and many others used per capita consumption to adjust the data for population growth. By taking all the expenditure per head the effect of variations in family size or variations in population can be isolated. Thus in this study the model is fitted both to the aggregate and per capita data for comparison.

The income and price elasticities of expenditure are calculated at average income, average relative price, and average expenditure levels. The elasticity of expenditure with respect to its own relative price is defined as the ratio of percentage change in expenditure to the percentage change in relative price of the i th commodity group. Thus the price elasticity at the average level is,

$$e_{i\bar{P}} = \partial \log E_i / \partial \log P_i = \frac{\bar{P}_i}{\bar{E}_i} \frac{\partial E_i}{\partial P_i}.$$

Similarly, the income elasticity of expenditure at the average levels for the i th commodity can be written as,

$$e_{i\bar{Y}} = \partial \log E_i / \partial \log Y = \frac{\bar{Y}}{\bar{E}_i} \frac{\partial E_i}{\partial Y}$$

Single equation multiple regression method of estimation is applied to each of the commodity groups. The estimated coefficients, the Durbin Watson statistic, the square of

multiple correlation coefficient R^2 are shown in the tables for each specification. The computed t-Statistic values are shown in the parentheses below each coefficient. The results are presented for both aggregate and per capita data in separate tables. The estimated coefficients or the computed t-Statistic values are not shown in the table either for the constant or the seasonal dummy variables which are not of much interest in this study.

It is convenient at this point to summarize the notation.

- E_{it} : Consumption expenditure on the i th commodity group at time t in constant 1957 dollars.
- Y_t : Real disposable income in constant 1957 dollars at time t .
- P_{it} : Relative price index of the i th commodity at time t .
- T : Residual time trend. $T=1,2,\dots,n$.
- S_1, S_2, S_3 : Quarterly seasonal dummy variables.'
- b, c : The coefficients of disposable income and relative price index.
- d_1, d_2, d_3 : The coefficients of the seasonal dummy variables.
- u_t : The disturbance term.
- E_{it}^* : The per capita consumption expenditure on the i th commodity at time t at constant 1957 dollars.
- Y_t^* : The per capita real disposable income at time t in constant dollars.
- $e_{\bar{p}}$: Price elasticity of expenditure at the average income and expenditure levels.

- $e_{\bar{y}}$: Income elasticity of expenditure at the average income and expenditure levels.
- DW : Durbin Watson statistic.
- R^2 : Square of the multiple correlation coefficient.

Specification 1.

The fixed coefficient model specified in equation (1) Chapter II can be written after introducing the seasonal quarterly dummy variables and the disturbance term as,

$$E_{it} = a_i + b_i Y_t + c_i P_{it} + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it}$$

for the i th commodity group. The corresponding specification in per capita terms for the i th commodity group is,

$$E_{it}^* = a_i + b_i Y_t^* + c_i P_{it} + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it}$$

Here i represents the four aggregated commodity groups Food, Clothing, Household goods, and Pleasure goods. The results of the two equations are reported in Tables 1 and 2.

The commodity group food showed normal behaviour with all the expected signs both for aggregate and per capita data, the coefficient of income is significantly different from zero. But the price variable is not significant. There is evidence of serial correlation among the disturbances. The elasticity of expenditure for food with respect

TABLE 1
STATISTICAL RESULTS AND ELASTICITIES OF EXPENDITURE FOR
SPECIFICATION 1 WHEN AGGREGATE DATA IS USED

Commodity	b^{\dagger}	c	R^2	DW	$e_{\bar{p}}$	$e_{\bar{y}}$
Food	14.4678 (19.3549)*	-732.8062 (-1.4277)	0.9486	0.9401	-0.5323	0.6552
Clothing	4.6521 (18.0697)	-247.0631 [†] (-2.0470)	0.9879	1.8246	-0.5024	0.5956
Household goods	3.5468 (3.6956)	-251.1587 (-1.0329)	0.9596	1.8637	-0.8356	0.7658
Pleasure goods	6.6967 (22.0127)	43.9039 (0.3114)	0.9742	1.7883	0.0825	0.7723

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 2

STATISTICAL RESULTS AND ELASTICITIES OF EXPENDITURE FOR
SPECIFICATION 1 WHEN PER CAPITA DATA IS USED

Commodity	b^{\dagger}	c	R^2	DW	$e_{\bar{p}}$	$e_{\bar{y}}$
Food	7.5662 (8.8016)*	-30.0605 (-1.6623)	0.9233	1.3117	-0.3939	0.3417
Clothing	2.0610 (6.0585)	-7.8653 (-1.6098)	0.9917	1.9494	-0.2887	0.2634
Household goods	2.8821 (2.6292)	-11.7768 (-1.3841)	0.9722	1.8834	-0.7082	0.6220
Pleasure goods	4.7641 (9.0548)	-3.0287 (-0.4054)	0.9620	1.9287	-0.1029	0.5489

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

to price and income at the average levels are less than unity supporting the hypothesis that food is a necessary commodity.

Clothing has all the significant coefficients with the appropriate signs in the aggregate data. The income coefficient was significant whereas price coefficient is not at the per capita level. The disturbances are free from autocorrelation. The income and price elasticities are less than one.

Household goods have a significant income coefficient but not the price coefficient even though it is negative. There is no serial correlation in the time series. The income elasticity and price elasticity of expenditure at the average levels are less than one.

Pleasure goods have a normal behaviour when per capita data is used. But in aggregate data the price coefficient showed an insignificant direct relationship with the consumption expenditure. Given the results at the aggregate level pleasure goods are inferior commodities.

All the elasticities are less than one except the price elasticity of expenditure at the aggregate level, which has a very low positive value. The elasticities of expenditure at the individual level are lower than at the aggregate level. In conclusion, income showed a significant direct relationship with expenditure for all the commodity groups. The relative price showed an inverse relation with the expenditure in all the commodity groups ex-

cept for pleasure goods when aggregate data were used. All the commodity groups have a high R^2 value. Overall, the results based on per capita data are more acceptable than the aggregate data. These results are in agreement with the result obtained by Reddy [21].

Specification 2

The fixed coefficient model specified in equation (2) in Chapter II with trend can be written in the stochastic form after introducing the seasonal dummy variables for the i th commodity group as,

$$E_{it} = a_i + b_i Y_t + c_i P_{it} + d_i T + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it}$$

The corresponding equation for the per capita expenditure on the i th commodity can be written as,

$$E_{it}^* = a_i + b_i Y_t^* + c_i P_{it} + d_i T + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it}$$

In this specification introduction of trend (residual trend) as an explanatory variable is expected to reflect the variations of consumption expenditure on any commodity group which are associated with the gradual changes in tastes and habits of the consumers over time. The estimated coefficients of the two equations are reported in Tables 3 and 4.

TABLE 3
STATISTICAL RESULTS AND ELASTICITIES OF EXPENDITURE FOR
SPECIFICATION 2 WHEN AGGREGATE DATA IS USED

Commodity	b	c	d	R ²	DW	e _p	e _y
Food	-0.0790 (-0.0476)*	-473.7131 (-1.6873)	10.2758 [†] (9.0461)	0.9852	1.5658	-0.3441	-0.0036
Clothing	1.6382 (1.4732)	-98.3868 (-0.8021)	2.2209 [†] (2.7730)	0.9902	1.7608	-0.2001	0.2098
Household goods	0.9661 (0.6092)	-215.4822 (-0.9216)	1.9416 (1.9978)	0.9639	2.0504	-0.7169	0.2086
Pleasure goods	0.7706 (0.7101)	-215.9610 (-1.9155)	4.2936 [†] (5.5784)	0.9867	1.5531	-0.4039	0.0889

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 4

STATISTICAL RESULTS AND ELASTICITIES OF EXPENDITURE FOR
SPECIFICATION 2 WHEN PER CAPITA DATA IS USED

Commodity	b	c	d	R ²	DW	e _P	e _Y
Food	1.6409 (1.0876)*	-32.3001 [†] (-2.2187)	0.1400 [†] (4.4188)	0.9518	1.7524	-0.4233	0.0741
Clothing	1.2143 (1.3430)	-4.9240 (-0.8660)	0.0207 (1.0106)	0.9920	1.8872	-0.1807	0.1552
Household goods	2.1163 (1.7994)	-3.3226 (-0.3362)	0.0414 (1.5867)	0.9741	2.0772	-0.2006	0.4568
Pleasure goods	0.2776 (0.2469)	-14.9032 [†] (-2.2379)	0.1062 [†] (4.3133)	0.9757	1.8330	-0.5062	0.0320

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

In the aggregate data, expenditure on commodity group food has an inverse relation with respect to relative price and disposable income. There is a significant effect of trend in the consumption expenditure on food. In the per capita data the relative price has a significant negative relationship with food consumption. Based on both aggregate and per capita data food is a necessary commodity with an income and price elasticities less than one.

Clothing and household goods can be classified as normal goods. The presence of trend is significant at the aggregate level. The other explanatory variables are not significant. The elasticities of expenditure with respect to relative price and disposable income are less than one for both commodity groups.

Even though the expenditure on Pleasure goods is not significantly explained by disposable income, a significant inverse relationship exists with relative price when per capita data is applied. Income elasticity of expenditure and price elasticity of expenditure are less than one with proper signs.

All the commodity groups exhibit independent disturbances over time with a high R^2 value.

The reason that many explanatory variables do not have statistically significant coefficients may be due to

the problem of multicollinearity.¹ Typically the correlation coefficient between real disposable income and trend is very high.²

The negative sign for the income coefficient in the aggregate data needs explanation. The negative income elasticity implies that food is an inferior good. The commodity group food consists of purchased foods, farm foods, and meals. The consumer expenditure on farm foods declined continuously over the period 1956-65 at the constant dollar. Thus farm foods are inferior goods. The inclusion of inferior commodity in food group might have resulted to a conclusion that food is an inferior good. The inclusion of trend has a significant positive effect on the expenditure of all commodities in aggregate data and two (food, pleasure goods) in case of per capita data. The consumer expenditure on these commodity groups is increasing along with the increasing tastes and habits of the consumers over time. This is not revealed by the specification 1.

¹ The problem of multicollinearity is discussed in [13], pp. 201-207.

² Holmes [10], p. 28; Houthakker [11], p. 285.

Specification 3

The dynamic form of the model that was given by equation (3) in Chapter II can be written for the i th commodity group as,

$$E_{it} = (a_i^* + a_i^{**}T) + (b_i^* + b_i^{**}T)Y_t + (c_i^* + c_i^{**}T)P_{it} \\ + d_1S_1 + d_2S_2 + d_3S_3 + u_{it}$$

or

$$E_{it} = a_i^* + a_i^{**}T + b_i^*Y_t + c_i^*P_{it} + b_i^{**}TY_t + c_i^{**}TP_{it} \\ + d_1S_1 + d_2S_2 + d_3S_3 + u_{it}$$

Similarly at the per capita level the stochastic form of the linear trend model for the i th commodity can be written as,

$$E_{it}^* = (a_i^* + a_i^{**}T) + (b_i^* + b_i^{**}T)Y_t^* + (c_i^* + c_i^{**}T)P_{it} \\ + d_1S_1 + d_2S_2 + d_3S_3 + u_{it}$$

or

$$E_{it}^* = a_i^* + a_i^{**}T + b_i^*Y_t^* + c_i^*P_{it} + b_i^{**}TY_t^* + c_i^{**}TP_{it} \\ + d_1S_1 + d_2S_2 + d_3S_3 + u_{it}$$

This is expected to represent the systematic changes in the expenditure due to changing tastes and habits. The results of these two equations are shown in Tables 5 to 8.

Food can be classified as a normal commodity. It can be noticed that the relative price index has a significant coefficient. The other explanatory variables including the composite trend variables do not have statistically significant coefficients.

Clothing does not exhibit normal behavior. None of the coefficients are significant except the composite trend coefficient of income. This implies that as the income increases the changes in tastes and habits of people increase the consumption expenditure.

All the explanatory variables have significant coefficient in the expenditure equation on household goods with one exception. The relative price does not have a significant coefficient when per capita data is used. The expenditure on household goods is directly related by the residual trend.

Results obtained by using aggregate data and per capita data, classify pleasure goods as inferior commodities. When aggregate data is applied only the composite trend coefficient of income is significant. The changing tastes and habits have a significant direct influence on expenditure on pleasure goods via the changes in income.

It is evident from the results that the four commodity groups are free from autocorrelation and R^2 value is high throughout.

TABLE 5

STATISTICAL RESULTS FOR SPECIFICATION 3 WHEN
AGGREGATE DATA IS USED

Commodity	a**	b*	c*	b**	c**	R ²	DW
Food	-18.8902 (-0.8388)*	1.4732 (0.5619)	-1069.9297 [†] (-2.0988)	-0.0511 (-1.2843)	31.9700 (1.3396)	0.9862	1.5242
Clothing	-5.5768 (-0.5861)	-1.8179 (-1.1992)	-280.2459 (-1.6716)	0.0678 [†] (3.0108)	4.8348 (0.5340)	0.9925	1.9405
Household goods	50.0414 [†] (5.1170)	7.1986 [†] (5.0036)	591.8728 [†] (2.6838)	-0.2022 [†] (-7.2166)	-38.1956 [†] (-4.5095)	0.9874	2.2438
Pleasure goods	3.4633 (0.3032)	-1.8399 (-1.1753)	-111.9803 (-0.4875)	0.0504 [†] (2.2908)	-1.3128 (-0.1191)	0.9889	1.6164

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 6

PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 3 WHEN AGGREGATE DATA IS USED

Time in Quarters	$e_{\bar{p}}$				$e_{\bar{y}}$			
	Food	Clothing	Household goods	Pleasure goods	Food	Clothing	Household goods	Pleasure goods
1	-0.7540	-0.5600	1.8420	-0.2130	0.0644	-0.2241	1.5109	-0.2064
10	-0.5448	-0.4716	0.6984	-0.2352	0.0436	-0.1460	1.1179	-0.1541
20	-0.3127	-0.3732	-0.5724	-0.2599	0.0204	-0.0592	0.6813	-0.0960
30	-0.0805	-0.2749	-1.9190	-0.2846	-0.0027	0.0277	0.2446	-0.0378
40	0.1517	-0.1766	-3.1138	-0.3092	-0.0258	0.1145	-0.1921	-0.0203

TABLE 7

STATISTICAL RESULTS FOR SPECIFICATION 3 WHEN
PER CAPITA DATA IS USED

Commodity	a**	b*	c*	b**	c**	R ²	DW
Food	-1.8673 (-1.6975)*	2.1666 (1.1371)	-71.7003 [†] (-2.8962)	-0.0661 (-1.5971)	2.2477 (1.9176)	0.9576	1.6502
Clothing	-0.0395 (-0.1046)	-1.0651 (-1.2274)	-6.6833 (-1.0815)	0.0765 [†] (4.3481)	-0.1979 (-0.5641)	0.9956	2.0761
Household goods	1.4630 [†] (3.8703)	4.8666 [†] (5.0164)	14.7748 (1.6705)	-0.1515 [†] (-6.7182)	-0.9219 [†] (-2.8632)	0.9902	2.1759
Pleasure goods	-0.3818 (-0.5152)	-1.3377 (-0.9325)	-17.0824 (-1.1769)	0.0529 (1.8417)	0.3074 (0.4408)	0.9781	1.9416

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 8
PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 3 WHEN PER CAPITA DATA IS USED

Time in Quarters	$e_{\bar{p}}$				$e_{\bar{y}}$			
	Food	Clothing	Household goods	Pleasure goods	Food	Clothing	Household goods	Pleasure goods
1	-0.9104	-0.2526	0.8331	-0.5698	0.0949	-0.1263	1.0176	-0.1480
10	-0.6453	-0.3179	0.3341	-0.4798	0.0680	-0.0384	0.7234	-0.0932
20	-0.3508	-0.3906	-0.2203	-0.3714	0.0382	0.0590	0.3964	-0.0322
30	-0.0562	-0.4632	-0.7747	-0.2670	0.0083	0.1572	0.0694	0.0287
40	0.2383	-0.5358	-1.4347	-0.1626	-0.0216	0.2549	-0.2576	0.0897

In this specification the elasticities are calculated at five time periods and at the average levels. In general, the estimated elasticities of expenditure are less than one (in absolute value) except household goods supporting the hypothesis that the commodities under consideration are necessities. The elasticities change the sign in a particular period.

The linear trend model is not theoretically acceptable because many estimated coefficients are not statistically significant and do not possess the proper sign. This has been experienced by many investigators like Stone [29], Pollak and Wales [17], and Parks [16] who worked with linear trend models. In a situation like this one can expect the problem of multicollinearity due to the interdependence of explanatory variables. A new formulation with fixed coefficient to one of the explanatory variables and linear trend coefficient to the other explanatory variable is tried. This is expected to reduce the problem of multicollinearity and capture the systematic changes, if any, in one of the explanatory variables independent of the other.

Specification 4

In this specification one explanatory variable, income has variable coefficient and relative price has fixed coefficient. The functional form that is explained by equation (4) in Chapter II can be written for the i th com-

modity group as,

$$E_{it} = a_i + (b_i^* + b_i^{**}T)Y_t + c_i^*P_{it} + d_1S_1 + d_2S_2 \\ + d_3S_3 + u_{it}$$

or

$$E_{it} = a_i + b_i^*Y_t + b_i^{**}TY_t + c_i^*P_{it} + d_1S_1 + d_2S_2 \\ + d_3S_3 + u_{it}$$

For the per capita data the expenditure on the i th commodity group is,

$$E_{it}^* = a_i + (b_i^* + b_i^{**}T)Y_t^* + c_i^*P_{it} + d_1S_1 + d_2S_2 \\ + d_3S_3 + u_{it} \quad ,$$

or

$$E_{it}^* = a_i + b_i^*Y_t^* + b_i^{**}TY_t^* + c_i^*P_{it} + d_1S_1 + d_2S_2 \\ + d_3S_3 + u_{it}.$$

In this specification there is constant intercept. Changing tastes and habits have no influence on the expenditure via relative price. The systematic changes in tastes and

habits are expected to influence expenditure only via income. The results obtained by both aggregate data and per capita data are reported in Tables 9 to 12.

The consumer behaviour is similar regarding the expenditure on food and clothing, when aggregate data is used. The relative price and disposable income have inverse relation with the expenditure. The composite trend term is significant indicating that the changes in tastes and habits of the people increase the consumption expenditure on food and clothing as the income rises. But such a pattern is not noticed when per capita data is used. In this case food and clothing have a significant linear trend coefficient, and the relative price is significant in explaining the expenditure on food.

The expenditure on household goods is significantly explained by real disposable income in the relation. The changing tastes and habits have a significant direct influence on the expenditure on pleasure goods via the changes in income. The per capita data shows significance of relative price in the relation.

All the commodities have high R^2 value. There is no serial correlation among the disturbances for all the commodity groups except food which shows positive serial correlation.

The price elasticity of expenditure calculated at the average levels, which is constant, over time, has the expected sign and less than one for all the commodities.

TABLE 9
STATISTICAL RESULTS FOR SPECIFICATION 4 WHEN
AGGREGATE DATA IS USED

Commodity	b*	b**	c*	R ²	DW
Food	-1.7712 (-0.4558)*	0.1441 [†] (4.2317)	-431.4629 (-1.0141)	0.9667	1.0980
Clothing	-1.5124 (-1.0670)	0.0558 [†] (4.3965)	-188.4668 (-1.9193)	0.9924	1.9659
Household goods	5.4914 [†] (2.2753)	-0.0176 (-0.8788)	-255.4580 (-1.0469)	0.9605	1.8019
Pleasure goods	-2.2824 (-1.5100)	0.0810 [†] (6.0006)	-38.4229 (-0.3846)	0.9876	1.6720

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 10
PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 4 WHEN AGGREGATE DATA IS USED

Elasticity	Time in Quarters	Food	Clothing	Household goods	Pleasure goods
Income	1	-0.0737	-0.1865	1.1821	-0.2539
	10	-0.0150	-0.1222	1.1479	-0.1698
	20	0.0503	-0.0508	1.1099	-0.0764
	30	0.1156	0.0207	1.0719	0.0170
	40	0.1809	0.0922	1.0339	0.1105
Price		-0.3134	-0.3832	-0.8499	-0.0722

TABLE 11
STATISTICAL RESULTS FOR SPECIFICATION 4 WHEN
PER CAPITA DATA IS USED

Commodity	b*	b**	c*	R ²	DW
Food	0.9696 (0.5311)*	0.0384 [†] (3.9315)	-31.5404 [†] (-2.0815)	0.9478	1.7791
Clothing	0.0766 (0.0776)	0.0118 [†] (3.1396)	-2.8117 (-0.5386)	0.9927	1.8038
Household goods	2.6767 (1.9994)	0.0023 (2.1296)	-10.2771 (-1.0174)	0.9722	1.9232
Pleasure goods	-0.8193 (-0.6677)	0.0325 [†] (4.8261)	-12.9975 [†] (-2.1092)	0.9777	1.8824

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 12

PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 4 WHEN PER CAPITA DATA IS USED

Elasticity	Time in Quarters	Food	Clothing	Household goods	Pleasure goods
Income	1	0.0455	0.0113	0.5769	-0.0907
	10	0.0612	0.0249	0.5814	-0.0570
	20	0.0785	0.0399	0.5863	-0.0195
	30	0.0959	0.0550	0.5913	0.0179
	40	0.1132	0.0701	0.5963	0.0554
Price		-0.4133	-0.1032	-0.6180	-0.4415

The per capita data show a positive income elasticity for all commodity groups except pleasure goods. The income elasticity of expenditure on pleasure goods attains zero at a certain period. The aggregate data exhibits a mixed pattern of income elasticities. Household goods are luxuries. The other three commodity groups have zero income elasticities at certain periods of time.

The linear trend model with fixed coefficient to one of the explanatory variables is better represented at individual level than at aggregate level.

Specification 5

In this specification relative price has a variable coefficient and real disposable income the fixed coefficient. The functional form defined in equation (5) in Chapter II can be written for the i th commodity group as,

$$E_{it} = a_i + b_i^* Y_t + (c_i^* + c_i^{**} T) P_{it} + d_1 S_1 + d_2 S_2 \\ + d_3 S_3 + u_{it}$$

or

$$E_{it} = a_i + b_i^* Y_t + c_i^* P_{it} + c_i^{**} T P_{it} + d_1 S_1 + d_2 S_2 \\ + d_3 S_3 + u_{it}$$

The same relation at the individual level can be written,

$$E_{it}^* = a_i + b_i^* Y_t^* + (c_i^* + c_i^{**} T) P_{it} + d_1 S_1 + d_2 S_2 \\ + d_3 S_3 + u_{it}$$

or

$$E_{it}^* = a_i + b_i^* Y_t^* + c_i^* P_{it} + c_i^{**} T P_{it} + d_1 S_1 + d_2 S_2 \\ + d_3 S_3 + u_{it}$$

In this specification the changing tastes and habits have no influence on expenditure via income, but are expected to influence via relative price. The results are shown in Tables 13 to 16.

From the results based on aggregate data, the expenditure on food is inversely related with disposable income. The relative price index and composite trend have significant coefficients. The same thing is true when per capita data is used except that the income has direct influence on expenditure.

The expenditure on clothing and household goods are explained in a similar way by both aggregate data and per capita data. In the aggregate data the composite trend has significant influence on the expenditure. In per capita data none of the coefficients are statistically significant. Both aggregate and per capita data exhibit the significance of relative price and composite trend in explaining the expenditure on pleasure goods.

TABLE 13
STATISTICAL RESULTS FOR SPECIFICATION 5 WHEN
AGGREGATE DATA IS USED

Commodity	b*	c*	c**†	R ²	DW
Food	-0.5929 (-0.3500)*	-651.3721† (-2.3508)	10.5841 (9.1538)	0.9855	1.5706
Clothing	1.7320 (1.5630)	-126.9729 (-1.0626)	2.1594 (2.6974)	0.9901	1.7488
Household goods	1.1507 (0.7868)	-249.8012 (-1.0776)	1.9121 (2.1003)	0.9644	2.0719
Pleasure goods	0.8166 (0.7405)	-292.8865† (-2.4229)	4.2119 (5.4457)	0.9864	1.5663

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 14

PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 5 WHEN AGGREGATE DATA IS USED

Elasticity	Time in Quarters	Food	Clothing	Household goods	Pleasure goods
Price	1	-0.4655	-0.2538	-0.8247	-0.5427
	10	-0.3963	-0.2143	-0.7675	-0.3922
	20	-0.3195	-0.1704	-0.7038	-0.2339
	30	-0.2425	-0.1265	-0.6402	-0.4714
	40	-0.1656	-0.0826	-0.5766	-0.3131
Income		-0.0269	0.2218	0.2485	0.0942

TABLE 15
STATISTICAL RESULTS FOR SPECIFICATION 5 WHEN
PER CAPITA DATA IS USED

Commodity	b*	c*	c**	R ²	DW
Food	1.4881 (0.9776)*	-34.7603† (-2.3946)	0.1426† (4.4769)	0.9526	1.7547
Clothing	1.2939 (1.4292)	-5.4096 (-0.9686)	0.0188 (0.9148)	0.9920	1.8959
Household goods	2.1782 (1.9028)	-3.6412 (-0.3806)	0.0438 (1.6980)	0.9744	2.0997
Pleasure goods	0.2608 (0.2303)	-16.9102† (-2.4585)	0.1053† (4.2966)	0.9756	1.8352

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 16

PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 5 WHEN PER CAPITA DATA IS USED

Elasticity	Time in Quarters	Food	Clothing	Household goods	Pleasure goods
Price	1	-0.4536	-0.1979	-0.2163	-0.5708
	10	-0.4368	-0.1917	-0.1926	-0.5386
	20	-0.4181	-0.1848	-0.1663	-0.5028
	30	-0.3995	-0.1779	-0.1400	-0.4671
	40	-0.3808	-0.1710	-0.1136	-0.4313
Income	.	0.0672	0.1654	0.4701	0.0300

There is no presence of serial correlation for all the commodity groups. The square of the multiple correlation coefficient is very high indicating very little unexplained variation by this specification.

The income elasticity of expenditure, at the average levels, which is constant, is positive and less than one for all commodity groups except food when aggregate data is used. The price elasticity of expenditure, which is dependent on time, has the desired sign.

The model with linear trend coefficient for relative price and fixed coefficient for disposable income is reasonably good. The changes in tastes and habits are significantly reflected in the consumption expenditures via the changes in relative prices for all commodity groups when aggregate data is used and for two when per capita data is used. Undoubtedly the per capita data performs better in explaining the consumption expenditures than the aggregate data.

The results obtained by the linear trend model with variable coefficients on one of the explanatory variable is better off than the one with variable coefficients on both the explanatory variables. In particular the model with linear trend coefficient on relative price is superior than the other two. An improvement is noticed by applying per capita data than the aggregate data in all the three specifications.

Specification 6

The habit formation model that was specified by the equation (6) in Chapter II can be written for the i th commodity group as,

$$\begin{aligned} E_{it} = & (a_i^* + a_i^{**} E_{it-1}) + (b_i^* + b_i^{**} E_{it-1}) Y_t \\ & + (c_i^* + c_i^{**} E_{it-1}) P_{it} \\ & + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it} \end{aligned}$$

or

$$\begin{aligned} E_{it} = & a_i^* + a_i^{**} E_{it-1} + b_i^* Y_t + c_i^* P_{it} + b_i^{**} E_{it-1} Y_t \\ & + c_i^{**} E_{it-1} P_{it} + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it}. \end{aligned}$$

The corresponding per capita expenditure for the i th commodity group can be written as,

$$\begin{aligned} E_{it}^* = & (a_i^* + a_i^{**} E_{it-1}^*) + (b_i^* + b_i^{**} E_{it-1}^*) Y_t^* \\ & + (c_i^* + c_i^{**} E_{it-1}^*) P_{it} \\ & + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it} \end{aligned}$$

or

$$E_{it}^* = a_i^* + a_i^{**} E_{it-1}^* + b_i^* Y_t^* + c_i^* P_{it} + b_i^{**} E_{it-1}^* Y_t^* \\ + c_i^{**} E_{it-1}^* P_{it} + d_1 S_1 + d_2 S_2 + d_3 S_3 + u_{it}.$$

In this specification the variable intercept is a function of past consumption expenditure. This also explains the dependence of present expenditure on the one period lagged consumption expenditure. The results of these two equations are reported in Tables 17 to 20.

The food equation does not have any significant coefficients. The income coefficient is negative when per capita data is applied.

Clothing has three significant coefficients but the sign of relative price is positive. Income has a significant coefficient when aggregate data is used. According to the results based on aggregate data, we can say that due to the consumption habits and tastes, the expenditure on clothing will decrease as income rises. The lagged expenditure has a significant direct influence on present consumption when per capita data is used.

The two components of the linear lagged consumption coefficient for relative price of household goods are not significant when per capita data is used. All the other coefficients are significant. The price coefficient is positive.

Pleasure goods have normal behavior as far as the

TABLE 17

STATISTICAL RESULTS FOR SPECIFICATION 6 WHEN
AGGREGATE DATA IS USED

Commodity	a**	b*	c*	b**	c**	R ²	DW
Food	-1.0053 (-0.3697)*	5.8462 (0.7197)	-2997.5295 (-0.0799)	-0.0016 (-0.3215)	1.8625 (0.6473)	0.9774	2.3672
Clothing	2.2479 (1.7392)	6.6053 [†] (4.7969)	410.2244 (0.7372)	-0.0066 [†] (-2.5830)	-1.5322 (-1.1677)	0.9920	2.3555
Household goods	9.3006 [†] (2.8079)	13.8602 [†] (3.8844)	1843.3474 [†] (2.2056)	-0.0387 [†] (-3.2043)	-6.8662 [†] (-2.5209)	0.9730	2.0039
Pleasure goods	1.1413 (0.7022)	7.6225 [†] (3.2131)	-87.8278 (0.1018)	-0.0077 [†] (-2.2394)	-0.1213 (-0.0733)	0.9841	2.2332

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 18

PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 6 WHEN AGGREGATE DATA IS USED

Time in Quarters	$e_{\bar{p}}$				$e_{\bar{y}}$			
	Food	Clothing	Household goods	Pleasure goods	Food	Clothing	Household goods	Pleasure goods
2	-0.7073	-0.2161	-0.1972	-0.2531	0.1861	0.5610	0.6773	0.5366
11	-0.5033	-0.5286	1.0774	-0.2726	0.1751	0.4766	1.1437	0.4607
21	-0.2269	-1.1829	-1.3416	-0.3090	0.1603	0.2987	0.2586	0.3188
31	-0.1916	-0.7567	-0.3228	-0.2955	0.1584	0.4143	0.6314	0.3710
41	0.0737	-1.4705	-3.3244	-0.3372	0.1442	0.2207	-0.4668	0.2089

TABLE 19

STATISTICAL RESULTS FOR SPECIFICATION 6 WHEN
PER CAPITA DATA IS USED

Commodity	a**	b*	c*	b**	c**	R ²	DW
Food	-5.3412 (-1.1560)*	-0.1034 (-0.0059)	-449.2534 (-1.2362)	0.0456 (0.1998)	5.6817 (1.1379)	0.9368	2.0639
Clothing	2.5739 [†] (2.0336)	2.8722 (1.4148)	49.0954 (1.5460)	-0.0189 (-0.2682)	-2.5492 (-1.9579)	0.9932	2.0929
Household goods	6.0557 [†] (2.0331)	12.8829 [†] (2.6274)	52.1799 (1.5334)	-0.6535 [†] (2.0945)	-3.7608 (-1.8205)	0.9775	1.8487
Pleasure goods	0.4441 (0.2012)	3.9785 (0.8055)	-5.1536 (-0.0785)	-0.0259 (-0.1697)	0.0170 (0.0075)	0.9657	2.0982

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 20

PRICE AND INCOME ELASTICITIES OF EXPENDITURE AT THE AVERAGE LEVELS
FOR SPECIFICATION 6 WHEN PER CAPITA DATA IS USED

Time in Quarters	$e_{\bar{p}}$				$e_{\bar{y}}$			
	Food	Clothing	Household goods	Pleasure goods	Food	Clothing	Household goods	Pleasure goods
2	-0.8276	-0.1705	-0.7806	-0.1611	0.1353	0.3161	0.3363	0.3864
11	-0.4931	-0.5941	-0.2076	-0.1591	0.1446	0.3050	0.9530	0.3761
21	0.0597	-1.5539	-0.9618	-0.1549	0.1599	0.2804	0.2237	0.3541
31	-0.1049	-0.7258	-0.2437	-0.1576	0.1553	0.3018	0.6715	0.3680
41	0.3887	-1.7041	-1.6053	0.1530	0.1690	0.2765	-0.0832	0.3443

signs are concerned. The two components of the linear lagged consumption coefficient of income are significant when aggregate data is used. The other explanatory variables do not display significant influence on expenditures.

All the commodities are free from autocorrelation. High R^2 value is also noticed for all the commodity groups.

Here the elasticities of expenditure are functions of previous expenditure. All the commodities have positive income elasticity and are less than one. The only exception to this is household goods in the eleventh quarter. The price elasticity of expenditure on food is negative and less than one in all quarters except one. Clothing has negative price elasticity consistently. The price elasticity of expenditure of household goods is positive in certain time period. Similarly the pleasure goods have a positive price elasticity in the last quarter.

From the results based on both aggregate and per capita data the habit formation model is not satisfactory. Many estimated coefficients are not significant and do not possess proper signs. But very high R^2 values are noticed with independent disturbances over time.¹ This suggests that there could be certain systematic relationships among the explanatory variables leading to the problem of multi-

¹ When the lagged dependent variable appears as an explanatory variable ordinary least squares method of estimation is biased but consistent. [13], pp. 211-220.

collinearity. This is likely due to the functional form of the variable coefficient model.

Specification 7

The proportional habit model set up in equation (7) in Chapter II can be written for the i th commodity group as,

$$E_{it} = a_i E_{it-1} + b_i E_{it-1} Y_t + c_i E_{it-1} P_{it} + d_1 S_1 \\ + d_2 S_2 + d_3 S_3 + u_{it}.$$

In terms of per capita expenditure the same equation can be written as,

$$E_{it}^* = a_i E_{it-1}^* + b_i E_{it-1}^* Y_t^* + c_i E_{it-1}^* P_{it} + d_1 S_1 \\ + d_2 S_2 + d_3 S_3 + u_{it}.$$

In this specification the expenditure on any commodity group is not only a function of relative price index and real disposable income but also a function of lagged consumption expenditure. This is expected to represent the consumers' changing tastes and habits over time directly. The results obtained by using aggregate and per capita data are shown in Tables 21 to 24.

TABLE 21
STATISTICAL RESULTS FOR SPECIFICATION 7 WHEN
AGGREGATE DATA IS USED

Commodity	a	b	c	R ²	DW
Food	1.0920 [†] (3.9218)*	0.0023 [†] (2.3647)	-0.4297 (-1.5361)	0.9767	2.5067
Clothing	1.3158 [†] (3.4235)	0.0035 (1.9524)	-0.9210 [†] (-2.4433)	0.9858	2.8437
Household goods	0.5213 (0.4333)	0.0085 (1.6848)	-0.9511 (-1.0176)	0.9482	1.8493
Pleasure goods	0.9223 [†] (2.3293)	0.0022 (1.3879)	-0.2199 (-0.8531)	0.9786	2.0336

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 22
PRICE AND INCOME ELASTICITIES OF EXPENDITURES AT THE AVERAGE LEVELS
FOR SPECIFICATION 7 WHEN AGGREGATE DATA IS USED

Time in Quarters	$e_{\bar{p}}$				$e_{\bar{y}}$			
	Food	Clothing	Household goods	Pleasure goods	Food	Clothing	Household goods	Pleasure goods
2	-0.3392	-0.6314	-0.8768	-0.1595	0.1122	0.1520	0.5063	0.0979
11	-0.3862	-0.8192	-0.7002	-0.1948	0.1278	0.1972	0.4043	0.1196
21	-0.4395	-1.2125	-1.0353	-0.2609	0.1486	0.2921	0.5978	0.1601
31	-0.4582	-0.9563	-0.8942	-0.2365	0.1516	0.2302	0.5163	0.1452
41	-0.5194	-1.3854	-1.3100	-0.3120	0.1726	0.3334	0.7564	0.1915

TABLE 23
STATISTICAL RESULTS FOR SPECIFICATION 7 WHEN
PER CAPITA DATA IS USED

Commodity	a	b [†]	c	R ²	DW
Food	0.6800 [†] (2.4279)*	0.0612 (3.6339)	-0.4685 (-1.9173)	0.9327	2.0257
Clothing	0.4979 (1.6182)	0.0693 (3.8995)	-0.6340 [†] (-2.7771)	0.9922	2.0875
Household goods	0.0763 (0.7882)	0.1725 (2.1854)	-0.7756 (-1.3609)	0.9704	1.8714
Pleasure goods	0.2580 (0.7299)	0.0943 (3.4504)	-0.1688 (-0.6770)	0.9650	1.9783

* Numbers in the parentheses are the computed t-Statistic values.

† Significant at 5% level.

TABLE 24

PRICE AND INCOME ELASTICITIES OF EXPENDITURES AT THE AVERAGE LEVELS
FOR SPECIFICATION 7 WHEN PER CAPITA DATA IS USED

Time in Quarters	$e_{\bar{p}}$				$e_{\bar{y}}$			
	Food	Clothing	Household goods	Pleasure goods	Food	Clothing	Household goods	Pleasure goods
2	-0.4172	-0.4906	-0.8083	-0.1383	0.1871	0.1827	0.6452	0.2621
11	-0.4448	-0.5959	-0.6043	-0.1582	0.2003	0.2268	0.4824	0.2997
21	-0.4904	-0.8346	-0.8455	-0.2004	0.2209	0.3176	0.6749	0.3798
31	-0.4768	-0.6286	-0.6974	-0.1736	0.2147	0.2393	0.5567	0.3289
41	-0.5175	-0.8720	-0.9782	-0.2192	0.2331	0.3319	0.7808	0.4154

From the tables it can be seen that the expenditure on food is significantly explained by disposable income. There is a significant presence of lagged consumption expenditure in explaining the present consumption. A decrease in real disposable income will decrease the consumption expenditure on food in the present period when the changes in tastes, habits, preferences are taken into account.

The expenditure on clothing is reasonably explained by the explanatory variables. All the explanatory variables show a significant presence in explaining the expenditure on clothing except the lagged expenditure when per capita data is used.

Household goods do not show any significant dependence on the explanatory variables even though they have the desired signs. In case of per capita data income has a significant habit formation coefficient.

The expenditure on pleasure goods is significantly explained by the lagged expenditure at the aggregate level and by the disposable income at the individual level.

The square of multiple correlation coefficient is high for all the commodity groups. There is evidence to believe that the commodity groups except clothing have independent disturbances over time. There could be negative serial correlation in the time series on clothing at aggregate level.¹

¹ There is, however, a problem in interpreting the DW Statistic with a lagged dependent variable.

The price elasticity of expenditure for all commodity groups evaluated at the average level of consumption and relative price have the desired sign. Similarly the income elasticity of expenditure for all commodity groups are positive as expected and can undoubtedly be classified as necessities.

The results obtained by the proportional habit model are more than satisfactory both at aggregate and per capita level. Thus the proportional habit model is strongly supported by the data and can be considered as an improvement on the classical static model. Real disposable income is significantly present in explaining the expenditure on all commodity groups.

The residual trend was introduced as an extension of the proportional habit formation model, but no improvement was noticed in results.

CHAPTER IV

LINEAR EXPENDITURE SYSTEM

The Estimation of the Parameters

In a linear expenditure system the expenditures on individual commodities are expressed as linear functions of total expenditure and prices. The demand functions introduced in equation (9) in Chapter II are reproduced here. The quantity demanded of the i th commodity can be written as,

$$X_i = c_i + \frac{b_i}{P_i} (Y - \sum_j P_j c_j) \quad i=1,2,\dots,m$$

The corresponding expenditure function for the i th commodity group is,

$$E_i = P_i X_i = P_i c_i + b_i (Y - \sum_j P_j c_j) \quad (1)$$

This can also be written as,

$$E_i = b_i Y + \sum_{j=1}^m c_j (\delta_{ij} - b_i) P_j \quad (1')$$

where $\delta_{ij} = 0$ for $i \neq j$
 $= 1$ for $i = j$

Recall that: E_i = Expenditure on the i th commodity group
in constant 1957 dollars per head.

P_i = Price index of the i th commodity group.

Y = Total expenditure on all non-durable goods at constant dollars per head.

$$= \sum_i E_i$$

Stone [25] recognized an important feature of this model, namely we can estimate the parameters in it by reasonably plausible and efficient methods. In doing so the complication of supply equations is ignored and an iterative two stage method based on minimizing the sum of the squares of the residuals in each equation at each point of time is adopted. This enables us to use the method of least squares with the usual scalar variance matrix for the disturbances. This is the procedure followed here and was also attempted by Stone [22]. This method is appealing because it is simple and straight forward generalization of single equation ordinary least squares. But its major criticism is that, the procedure is not based on any specification of the error structure [16,17]. The distribution of the estimators is not known and hence no meaningful standard errors can be obtained by this method. No tests of hypothesis are possible.

The expenditure equation for the commodity groups can be written in the matrix form as,

$$E \equiv PX \equiv Pc + b(Y - P'c) \quad (2)$$

where,

$$P \equiv \begin{bmatrix} P_1 & 0 & \dots & 0 \\ 0 & P_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & P_m \end{bmatrix} \quad \text{and} \quad X \equiv \begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_m \end{bmatrix}$$

b and c are vectors of constants,

$$b' = (b_1, b_2, \dots, b_m) \quad \text{and} \quad c' = (c_1, c_2, \dots, c_m)$$

thus,

$$PX \equiv \begin{bmatrix} P_1 X_1 \\ P_2 X_2 \\ \dots \\ P_m X_m \end{bmatrix}, \quad P'c = (P_1 c_1 + P_2 c_2 + \dots + P_m c_m) = \sum_j P_j c_j$$

The stochastic form of the equation (1) incorporating the entire set of observations on the i th commodity can be written as,

$$E_i = P_i c_i + b_i (Y - \sum_j P_j c_j) + u_i \quad (3)$$

where u_i is the disturbance term of n observations.

Here, $E(u_i^2) = \sigma^2$ and $E(u_i) = 0$

have to be estimated simultaneously from the above set of equations. To solve this, equation (4) after substituting the estimates for b_i can be written as,

$$W_i = Z_i c + u_i$$

where $Z_i = (\delta_{ij} - \hat{b}_i) P_j$ and $W_i = (E_i - \hat{b}_i Y)$

Z_i and W_i are vectors of n observations.

The system of equations (5) for the m commodity group over n time periods can be written as,

$$W = Zc + u$$

where,

$$W \equiv \begin{bmatrix} W_1 \\ W_2 \\ \dots \\ W_m \end{bmatrix}, \quad Z \equiv \begin{bmatrix} Z_1 \\ Z_2 \\ \dots \\ Z_m \end{bmatrix}, \quad u \equiv \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_m \end{bmatrix}$$

The ordinary least squares estimate \hat{c} of c is given by,

$$\hat{c} = (Z'Z)^{-1} Z'W$$

With this estimate of c it is now possible to write

$$Y^* = \sum_j P_j \hat{c}_j$$

and

$$E_i^* = P_i \hat{c}_i + b_i (Y - Y^*)$$

or

$$(E_i^* - P_i \hat{c}_i) = b_i (Y - Y^*)$$

Thus the stochastic equation (3) for the i th commodity after substituting the estimator of c can be written as,

$$M_i = N b_i + v_i$$

where

$$M_i = E_i^* - P_i \hat{c}_i$$

$$N = Y - Y^*$$

M_i and N are vectors of n observations. While estimating b the set of equations of each commodity are treated as independent of each other. Thus the vector b can be estimated by taking the least square regression of M_i on N in each of the commodities. The estimate of b_i is

$$\hat{b}_i = (N'N)^{-1} N'M_i \quad i=1,2,\dots,m$$

Given this estimate of b it is now possible to re-estimate c and to continue the process until stable estimates of b and c are reached. It is of course possible that it is neither "easy to obtain complete convergence in a case like this nor even to be certain that unique limit of convergence exists independently of the initial values

for b ".¹ The standard errors computed by this method are of no use for interpretation.

Empirical Results

In this section the results are given for the analysis of a system of four commodity groups among which the total expenditure on all non-durable goods per capita has been divided. The data used here is the same as was used in Chapter III with the same classification.

Initially the coefficients \hat{b} are estimated from the naive model. The estimated coefficients are substituted in equation (4) and \hat{c} are estimated. The estimated \hat{c} is substituted back in equation (3) and \hat{b} is obtained. This is continued till the 3rd iteration. The results of the iteration along with the differences are shown in Table 25. It has been noticed that the estimated coefficients are steadily decreasing. The process is terminated at the 3rd iteration because the changes in the estimated values for b between the second and third iteration is low.

It can be noticed from the final results at the end of the third iteration in Table 25, that all the estimated values of b are positive and less than one, which as expected since the b_i are marginal budget shares. The largest marginal budget share is noticed for food and the lowest for household goods. The b 's also satisfy the adding up

¹ Stone et al. [29], p. 207.

TABLE 25

RESULTS OF THE LINEAR EXPENDITURE SYSTEM

Iteration (1)

Commodity	b	c
Food	0.4706	0.0314
Clothing	0.2096	-0.0531
Household goods	0.1104	-0.0115
Pleasure goods	0.2095	-0.0238

Iteration (2)

Commodity	b	c
Food	0.4356(0.035)*	0.0186(0.013)
Clothing	0.2344(0.025)	-0.1252(0.072)
Household goods	0.1132(0.003)	-0.0334(0.022)
Pleasure goods	0.2169(0.008)	-0.0664(0.043)

* Numbers in the parentheses correspond to the difference from the previous iteration.

Iteration (3)

Commodity	b	c	$e_{\bar{p}}$	$e_{\bar{y}}$
Food	0.4068(0.029)*	0.0402(0.022)	0.0336	0.9337
Clothing	0.2548(0.021)	-0.1748(0.050)	-0.3380	1.0878
Household goods	0.1153(0.002)	-0.0445(0.011)	-0.2047	1.0178
Pleasure goods	0.2231(0.006)	-0.0898(0.024)	-0.2008	1.0292

* Difference from the previous iteration.

criterion, i.e., $\sum b_i = 1$. All the estimated values of c do not agree with theory. The committed quantity of food, c_1 is negative. The other commodities have a negative price coefficient. The calculated consumption values based on the estimated c 's are all less than the actual expenditures leaving a positive supernumerary income.

The elasticity of expenditure on the commodity group i with respect to the total expenditure on all non-durable goods can be written as,

$$e_Y = \frac{\partial \log E_i}{\partial \log Y} = \frac{Y}{E_i} \frac{\partial E_i}{\partial Y} = \frac{Y}{E_i} b_i \quad \text{from equation (1).}$$

At the average expenditure \bar{E}_i and total expenditure \bar{Y} , the elasticity can be written as,

$$e_{\bar{Y}} = b_i \frac{\bar{Y}}{\bar{E}_i}$$

Thus the elasticity of expenditure on all non-durable goods at the average levels is equal to the proportion of supernumerary income spent on the i th good divided by the proportion of average total expenditure devoted to the i th good. Similarly the price elasticity of expenditure for the i th commodity can be written as,

$$e_{P_i} = \frac{\partial \log E_i}{\partial \log P_i} = \frac{P_i}{E_i} \frac{\partial E_i}{\partial P_i} = \frac{P_i}{E_i} (1 - b_i) c_i$$

since $\frac{\partial E_i}{\partial P_i} = (1-b_i)c_i$ from equation (1).

At the average level of expenditure and price, the price elasticity equal to,

$$e_{\bar{P}_i} = (1-b_i)c_i \bar{P}_i/\bar{E}_i$$

Since all b 's are positive and total expenditure on all non-durable goods is greater than the committed expenditures, it can be described that the individual as purchasing necessary quantities of the various goods (c_1, c_2, \dots, c_4) and then dividing his supernumerary income ($Y - \sum_{j=1}^4 P_j c_j$) among the goods in fixed proportion (b_1, b_2, \dots, b_4).

The elasticity of expenditure for each commodity group with respect to the total expenditure on all non-durable goods is calculated at the average levels. From Table 25 it can be seen that these elasticities are positive and nearly equal to one for all the commodity groups. The elasticity of expenditure on food with respect to the total expenditure is 0.93 and that of pleasure goods is 1.03. This result has implications for the utility function. It has been noticed by Yoshihara [32] that the total expenditure elasticities are unity if and only if the utility function is homogeneous. Thus empirically we can conclude that the linear expenditure system is consistent with a homogeneous utility function. We can also note the estimated values of c are very low, i.e., the committed quanti-

ties of expenditure are very low. This too implies that the utility function under consideration is homogeneous.¹

The price elasticity of expenditure evaluated at the average price and expenditure levels are reported in Table 25. All the commodities support the hypothesis of negative price elasticity except food. Food has a positive price elasticity. Food may be an inferior good, but Stone ruled out such a possibility under the theory of consumer behavior in linear expenditure system [27]. The expenditure on food is the sum of the expenditures on Farm foods, Purchased foods, and Meals. Farm foods consists of food produced and consumed on farms. This includes all types of home-grown products consumed by farm families and hired help, based on farm prices. The expenditure on farm foods per head was steadily decreasing during the period under

¹ The utility function corresponding to the linear expenditure system when c_i are zero can be written as,

$$U(X_1, X_2, \dots, X_m) = \prod_{i=1}^m X_i^{b_i}$$

$$U(KX_1, KX_2, \dots, KX_m) = \prod_{i=1}^m (KX_i)^{b_i} = K^{\sum b_i} \prod_{i=1}^m X_i^{b_i} \text{ for } K \neq 0$$

$$\text{But } \sum_{i=1}^m b_i = 1$$

$$\therefore U(KX_1, \dots, KX_m) = K \prod_{i=1}^m X_i^{b_i} = KU(X_1, X_2, \dots, X_m)$$

study while the per capita income was rising. This means that more and more consumers that were consuming farm foods are shifting to other types of luxurious foods. This may be due to the development of new tastes or habits over time which were not reflected in the system.

The price elasticity of expenditure of these four commodity groups are lower when compared with the elasticities obtained in Chapter III.

These considerations bring out an important limitation of the system. It is not likely to provide a full description of actual behavior when the committed quantities are negative. In such a case a reformulation of the system is suggested by Stone [22] which would enable to introduce the changes in tastes and habits into the system. The linear expenditure system with dynamic coefficients has been investigated by Stone [23,24,26,27,29].

CHAPTER V

CONCLUSIONS

The three econometric models set up in this thesis explored Canadian consumer expenditure behaviour. The modified traditional single equation demand models yield empirically acceptable explanations of demand for individual commodity groups. The linear expenditure system is a more realistic model and also yielded satisfactory results.

The fixed coefficient model describes consumer behaviour satisfactorily. In fact the introduction of trend in the fixed coefficient model does not improve the results but created the problem of multicollinearity.

The dynamic demand functions, which are expected to reflect the tastes and habits of people over time, are well described in this study. Even though the linear trend and habit formation models encounter the problem of multicollinearity, the proportional habit formation model gave the best results. This is an improvement over the fixed coefficient models explored by many investigators with lagged consumption expenditure as an explanatory variable. The introduction of a linear trend for the relative price coefficient rather than income is highly significant.

The linear expenditure system gives estimates which show unit income elasticities and low price elasticities

of expenditure.

As far as the modified traditional model is concerned, the fixed coefficient model is theoretically plausible. The proportional habit formation model is the best among the variable coefficient specifications.

Overall, the per capita data proved better than the aggregate data in each specification. The significance of trend is noticed in all the models. The income elasticities of expenditure are less than one for all the commodities in all models with few exceptions. This supports the hypothesis that the commodity groups under consideration are largely necessities. Based on this investigation the commodity group food can be classified as an inferior good. It is noticed from the results that both kinds of demand studies do not differ as far as the elasticities are concerned. The results obtained by this study are comparable with similar studies based on Canadian economy. The method of aggregation of commodity groups is comparable with D.B.S. data.

As an improvement, a more disaggregated model for each of the nineteen non-durable goods could explain better than the aggregate commodity groups. The fact that the linear trend and habit formation models did not yield significant parameter estimates with proper signs suggest that an attempt could be made for more general specifications of the functional form of the demand equations.

As an alternative a more sophisticated estimation

methods can be used. Particularly when the lagged dependent variable is introduced as an explanatory variable ordinary least squares method of estimation is not optimal. Instead of fixed coefficients in the linear expenditure system a habit formation model could be explored for the Canadian economy.

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APPENDIX I

THE DATA AND GROUPING

In this study nineteen non-durable goods are considered. The data were supplied by Mr. A.S. Foti, Chief, National Accounts Section, Dominion Bureau of Statistics, Ottawa. Quarterly data for the period 1956-65 at constant 1957 dollars were available. The data were seasonally unadjusted. The data covers the prices (price index), consumer expenditure (in millions of dollars) for each of the non-durable goods, personal disposable income defined as personal income less personal direct taxes in millions of dollars, population of Canada in millions of persons, and the consumer price index of all goods and services.

There are certain limitations in the data. (i) The data were not seasonally adjusted. (ii) The titles of the commodities are not well defined. In other words the goods covered under the commodity name are not clearly known. (iii) Data are not available for the consumer price index of all non-durable goods as such at 1957 base year. (iv) The sum of the expenditure on the individual nineteen non-durable goods does not add up to the consumer expenditure on all non-durable goods as a whole in constant dollar terms.

An attempt is made to take care of the seasonal adjust-

ments. The dummy variables S_1, S_2, S_3 which will take one in the corresponding quarter and zero elsewhere are introduced. The regression analysis is carried out on actual as well as seasonal dummy variables. There is no need for a fourth seasonal dummy relating to the fourth quarter, and if introduced leads to multicollinearity. As far as the details of the commodity groups are concerned no attempt is made to check this. However some details are available from the literature of Dominion Bureau of Statistics. Here the major attempt is to group the nineteen commodity groups further into four major commodity groups. Care is taken only in explaining the grouping of the commodities. With regards to the third limitation of the data, a weighted price index of the non-durable goods is calculated by using expenditure shares as weights. A value share W_i is defined as the ratio of the expenditure of i th commodity to total expenditure of all non-durable goods.

$$W_i = E_i / \sum_{i=1}^{19} E_i \quad i=1,2,\dots,19$$

where E_i is expenditure on the i th commodity and $\sum E_i$ the total expenditure on all non-durable goods. Thus the consumer price index of all non-durable goods = $\sum_{i=1}^{19} W_i P_i$.

P_i the consumer price index of the i th good.

Also $\sum_i W_i = 1$.

The calculated price index given above is comparable with the consumer price index of all non-durable goods at the 1949 base year issues by D.B.S. These price indices are adjusted to 1957 base year and then the comparison is made with the calculated price index. The calculated price index is very close to the reported price index which indicated that the method of weighing is reasonable here. A comparative table is presented in Table I-1.

The nineteen non-durable goods considered in this study does not exhaust the total expenditure on non-durable goods. There could be some non-durable goods that are not listed here and so there could be some more consumer expenditure on those goods. However, since the expenditure on the nineteen non-durable goods adds to 92% of the total expenditure on non-durable goods the analysis is carried out with these nineteen commodities only.

The nineteen non-durable goods that are considered in this study are:

- (1) Farm Foods.
- (2) Purchased Foods.
- (3) Meals.
- (4) Men's Clothing.
- (5) Women's Clothing.
- (6) Footwear.
- (7) Piece Goods.
- (8) Notions.
- (9) Armed Force Clothing.

TABLE I-1
COMPARISON OF ACTUAL PRICE INDEX WITH CALCULATED
PRICE INDEX FOR ALL NON-DURABLE GOODS, (1957=100),
SEASONALLY UNADJUSTED

Quarter Year	I	II	III	IV
1956	95.803 (96.394)*	95.919 (96.943)	98.025 (97.550)	99.660 (97.723)
1957	99.218 (99.804)	100.938 (99.978)	100.949 (100.238)	99.317 (100.267)
1958	101.499 (102.059)	103.633 (102.290)	102.880 (102.666)	102.080 (102.810)
1959	102.517 (102.492)	102.615 (102.666)	103.185 (102.897)	103.568 (102.753)
1960	102.827 (103.186)	103.502 (103.591)	103.944 (103.822)	104.292 (104.024)
1961	104.950 (104.458)	104.256 (104.602)	105.024 (104.689)	104.487 (104.747)
1962	104.684 (105.325)	104.395 (105.932)	105.768 (106.019)	106.907 (106.279)
1963	106.915 (107.637)	111.313 (108.100)	109.315 (108.215)	108.600 (108.389)
1964	108.580 (109.487)	109.448 (109.603)	112.554 (110.094)	110.000 (110.210)
1965	109.729 (111.250)	111.763 (111.944)	113.427 (112.464)	113.281 (112.435)

* Figures in the parentheses refer to the actual price index by D.B.S.

- (10) Household Supplies.
- (11) Soap and Cleaning Supplies.
- (12) Fuel.
- (13) Electricity.
- (14) Gas.
- (15) Hardware.
- (16) Tobacco Products.
- (17) Alcoholic Products.
- (18) Drugs and Cosmetics.
- (19) Newspapers and Magazines.

The grouping of the data is arbitrary but can be justified according to the likeness of the commodities. For example all food products or all commodities relating to food are grouped under the commodity group Food. Similarly the commodity group Clothing represents all goods used for wearing. In the case of Household goods, all household operating goods like cleaning and maintenance goods are grouped together. The final group named as Pleasure goods is not so homogeneous as the other three. The homogeneity here is that all those goods used for amusement or recreational purposes are put together. There could be a variety of ways for grouping the commodities. But this grouping is comparable with many of the investigators such as Prais and Houthakker [20], Parks [16], and Pollak and Wales [17].

Group (A) Food: Farm foods, purchased foods, and meals.

Group (B) Clothing: Men's, women's, and children's clothing, footwear, piece goods, notions, armed force clothing.

Group (C) Household goods: Household supplies, soaps and cleaning supplies, fuel, electricity, gas, and hardware.

Group (D) Pleasure goods: Tobacco products, alcoholic products, drugs and cosmetics and newspapers and magazines.

The aggregate expenditure and the aggregate price index of the four commodity groups under consideration are obtained by a modified method of Barten [3]. Since all the expenditures are at the constant dollars the aggregate expenditure is simply the sum of the expenditures of the commodities within that group. If E_i the expenditure on the i th good, $i=1,2,\dots,19$, in the order given above then,

$$\text{Aggregate expenditure on Food } E_A = \sum_{i=1}^3 E_i$$

$$\text{Aggregate expenditure on Clothing } E_B = \sum_{i=4}^9 E_i$$

$$\text{Aggregate expenditure on Household goods } E_C = \sum_{i=10}^{15} E_i$$

$$\text{Aggregate expenditure on Pleasure goods } E_D = \sum_{i=16}^{19} E_i$$

Let N be the population of Canada in millions of people, then the per capita expenditure is obtained by

dividing the consumer expenditure on a commodity with the population. Aggregations are done as before with per capita expenditure on each commodity.

Let W_i represent the value share of these nineteen non-durable goods as defined earlier. Write,

$$W_A = \sum_{i=1}^3 W_i$$

$$W_B = \sum_{i=4}^9 W_i$$

$$W_C = \sum_{i=10}^{15} W_i$$

$$W_D = \sum_{i=16}^{19} W_i$$

These are the aggregate value shares of the four commodity groups called as the average value share of the composite commodity group A, B, C, and D. For example W_A represents the proportion of expenditure on food out of all non-durable goods. Define,

$$w_{iA} = W_i / W_A \quad i=1,2,3.$$

$$w_{iB} = W_i / W_B \quad i=4,5,6,7,8,9.$$

$$w_{iC} = W_i / W_C \quad i=10,11,12,13,14,15.$$

$$w_{iD} = W_i / W_D \quad i=16,17,18,19.$$

w_{iA} represents the average share of food spent on commodity i say farm foods. These are the weights used in forming the aggregate price index of each group. Let P_i represent the price index of the i th commodity $i=1,2,\dots,19$, in the same order given above. Then,

$$\text{Aggregate price index of Food } P_A = \sum_{i=1}^3 w_{iA} P_i$$

$$\text{Aggregate price index of Clothing } P_B = \sum_{i=4}^9 w_{iB} P_i$$

$$\text{Aggregate price index of Household goods } P_C = \sum_{i=10}^{15} w_{iC} P_i$$

$$\text{Aggregate price index of Pleasure goods } P_D = \sum_{i=16}^{19} w_{iD} P_i$$

The constructed price indices are closely comparable with the price index of Food, Clothing, and Household goods reported by D.B.S. at 1949 base year adjusted for 1957 base. Thus the method of aggregation is justified. There is no price index for Pleasure goods. However, it should be noted that the contents of the commodity groups reported by D.B.S. are not known. But it is worth attempting a comparison with it. Comparison of actual (reported by D.B.S.) and calculated price indices are shown in Tables I-2, I-3, and I-4. In the Household commodity group there are some deviations of the calculated price index from the actual price index. This may be due to the classification

TABLE I-2
COMPARISON OF ACTUAL PRICE INDEX WITH CALCU-
LATED PRICE INDEX FOR FOOD, (1957=100),
SEASONALLY UNADJUSTED

Quarter Year	I	II	III	IV
1956	93.077 (94.545)*	93.377 (95.929)	97.442 (95.979)	99.243 (96.457)
1957	98.893 (100.000)	101.735 (100.235)	101.549 (99.746)	99.325 (100.000)
1958	101.964 (102.614)	105.677 (102.895)	104.001 (102.867)	102.372 (103.514)
1959	103.482 (102.108)	102.421 (102.474)	102.659 (102.052)	102.898 (101.827)
1960	102.162 (102.642)	102.890 (103.289)	103.542 (102.727)	104.205 (103.343)
1961	104.748 (104.245)	103.935 (104.245)	104.787 (104.723)	103.559 (105.060)
1962	105.098 (105.735)	105.459 (106.213)	107.380 (106.578)	107.284 (107.393)
1963	108.355 (109.249)	107.903 (109.502)	111.136 (110.261)	109.364 (110.626)
1964	110.170 (111.132)	110.317 (110.992)	112.563 (112.088)	110.344 (112.482)
1965	111.165 (113.410)	113.253 (114.253)	115.518 (115.406)	115.642 (115.490)

* Figures in the parentheses correspond to the actual price index reported by D.B.S.

TABLE I-3

COMPARISON OF ACTUAL PRICE INDEX WITH CALCULATED PRICE INDEX FOR CLOTHING, (1957=100),
SEASONALLY UNADJUSTED

Quarter Year	I	II	III	IV
1956	100.333 (100.099)*	100.500 (100.069)	100.212 (100.069)	100.130 (100.069)
1957	99.359 (99.669)	100.066 (99.700)	99.824 (100.284)	100.971 (100.345)
1958	100.338 (100.867)	101.208 (100.898)	100.992 (101.328)	101.624 (101.359)
1959	100.358 (100.990)	100.777 (100.929)	100.768 (101.543)	102.018 (101.605)
1960	100.997 (101.851)	101.891 (101.974)	101.393 (102.527)	102.761 (102.527)
1961	102.174 (103.571)	103.299 (103.725)	103.264 (103.848)	104.141 (103.756)
1962	102.666 (103.756)	103.725 (104.616)	103.455 (105.015)	106.484 (104.985)
1963	105.618 (106.429)	106.485 (107.258)	106.340 (107.535)	109.365 (107.688)
1964	108.527 (109.317)	109.433 (109.839)	109.512 (110.146)	111.276 (110.300)
1965	109.632 (110.099)	111.098 (111.775)	110.835 (112.205)	113.434 (112.358)

* Figures in the parentheses correspond to the actual price index reported by D.B.S.

TABLE I-4

COMPARISON OF ACTUAL PRICE INDEX WITH CALCULATED PRICE INDEX FOR HOUSEHOLD GOODS, (1957=100),
SEASONALLY UNADJUSTED

Quarter Year	I	II	III	IV
1956	97.795 (97.548)	97.717 (97.826)	97.705 (97.966)	98.618 (98.077)
1957	100.480 (99.749)	100.105 (99.832)	99.806 (100.195)	99.793 (100.222)
1958	99.938 (100.947)	99.679 (101.058)	99.327 (101.198)	99.146 (101.365)
1959	99.819 (102.368)	100.767 (102.507)	101.514 (102.674)	100.993 (102.786)
1960	100.667 (103.009)	101.856 (103.009)	102.550 (103.064)	100.723 (103.120)
1961	100.536 (102.869)	101.374 (102.814)	102.349 (102.758)	100.581 (102.814)
1962	100.322 (103.204)	101.323 (103.259)	102.599 (103.399)	100.930 (103.399)
1963	101.176 (103.965)	102.963 (103.622)	102.252 (103.482)	100.363 (103.455)
1964	100.298 (103.733)	102.091 (103.761)	103.229 (103.872)	110.787 (104.012)
1965	100.678 (104.541)	101.916 (104.485)	103.375 (104.653)	100.744 (104.680)

* Figures in the parentheses correspond to the actual price index reported by D.B.S.

of the commodity group. The group of Household goods contain not only the household supplies but many other house cleaning and maintenance commodities. But in the reported figures by D.B.S. this group correspond to only household supplies. The other two commodity groups have the price indices which are very close to the D.B.S. indices.

APPENDIX II

THE DATA

The aggregate data utilized in this thesis is given in this appendix. The Personal disposable income, consumer price index of all goods and services, and population of Canada are also shown here. The detailed expenditure and price index for fourteen non-durable goods are given in Reddy [21]. The figures for the other five non-durable goods are available from Professor A. Buse, Department of Economics, University of Alberta, Edmonton.

TABLE II-1
AGGREGATE CONSUMER EXPENDITURE ON FOOD
MILLIONS OF 1957 DOLLARS

<u>Quarter</u> <u>Year</u>	I	II	III	IV
1956	1086.599	1211.300	1244.699	1224.400
1957	1143.800	1227.899	1282.199	1314.899
1958	1175.200	1237.399	1305.099	1340.699
1959	1210.100	1320.999	1379.600	1403.000
1960	1285.000	1369.900	1437.500	1441.700
1961	1272.599	1403.100	1453.599	1462.199
1962	1348.899	1433.500	1475.099	1500.299
1963	1364.500	1467.799	1502.899	1537.399
1964	1415.000	1491.800	1551.800	1606.700
1965	1434.500	1543.399	1598.900	1663.900

TABLE II-2

AGGREGATE PRICE INDEX FOR FOOD (1957=100)

<u>Quarter</u> <u>Year</u>	I	II	III	IV
1956	93.0773	93.3768	97.4419	99.2431
1957	98.8928	101.7346	101.5489	99.3247
1958	101.9636	105.6768	104.0007	102.3721
1959	103.4917	102.4105	102.6594	102.8983
1960	102.1621	102.8899	103.5420	104.2053
1961	104.7476	103.9352	104.7872	103.5592
1962	105.0975	105.4590	107.3802	107.2940
1963	108.3551	107.9033	111.1358	109.3642
1964	110.1696	110.3170	112.5628	110.3438
1965	111.1651	113.2534	115.5183	115.6422

TABLE II-3
AGGREGATE CONSUMER EXPENDITURE ON CLOTHING
MILLIONS OF 1957 DOLLARS

<u>Quarter</u> <u>Year</u>	I	II	III	IV
1956	337.0996	417.6995	376.4993	576.4993
1957	345.2993	442.1995	394.5996	576.8994
1958	363.6990	437.3994	404.2991	606.1995
1959	377.3994	456.3997	417.8997	628.1997
1960	375.5996	483.8992	437.4993	647.3997
1961	397.4995	477.7996	449.0996	645.0989
1962	402.3994	503.0989	462.5996	659.7993
1963	400.3992	510.5991	477.1995	674.0991
1964	424.9990	508.8992	494.6990	703.7991
1965	422.0996	537.3989	521.4988	739.6990

TABLE II-4

AGGREGATE PRICE INDEX FOR CLOTHING (1957=100)

<u>Quarter Year</u>	I	II	III	IV
1956	100.3334	100.5003	100.2115	100.1301
1957	99.3591	100.0664	99.8244	100.9712
1958	100.3383	101.2078	100.9919	101.6239
1959	100.3584	100.7773	100.7682	102.0181
1960	100.9970	101.8911	101.3934	102.7606
1961	102.1742	103.2994	103.2636	104.1412
1962	102.6663	103.7251	103.4545	106.4839
1963	105.6177	106.4849	106.3400	109.3651
1964	108.5271	109.4328	109.5122	111.2757
1965	109.6319	111.0975	110.8351	113.4339

TABLE II-5

AGGREGATE CONSUMER EXPENDITURE ON HOUSEHOLD GOODS
MILLIONS OF 1957 DOLLARS

<u>Quarter Year</u>	I	II	III	IV
1956	277.0996	211.3999	185.3999	271.0996
1957	295.6995	212.9999	183.5999	285.6995
1958	306.7993	221.2999	189.9999	310.7996
1959	338.0994	236.9999	195.1999	321.5994
1960	348.7996	243.6999	197.2999	327.1997
1961	354.6995	263.0999	201.3999	333.1997
1962	400.9995	272.7996	208.5999	364.4998
1963	413.1997	282.5996	219.8999	369.8997
1964	410.2998	298.5999	223.6999	395.7996
1965	435.1995	313.4995	235.5999	413.9998

TABLE II-6

AGGREGATE PRICE INDEX FOR HOUSEHOLD GOODS

(1957=100)

<u>Quarter</u> <u>Year</u>	I	II	III	IV
1956	97.7947	97.7173	97.7051	98.6184
1957	100.4800	100.1054	99.8062	99.7928
1958	99.9378	99.6786	99.3270	99.1462
1959	99.8187	100.7668	101.5144	100.9926
1960	100.6667	101.8560	102.5498	100.7234
1961	100.5355	101.3739	102.3486	100.5813
1962	100.3217	101.3233	102.5987	100.9295
1963	101.1759	102.9629	102.2520	100.3626
1964	100.2984	102.0911	103.2285	100.7872
1965	100.6779	101.9162	103.3748	100.7440

TABLE II-7

AGGREGATE CONSUMER EXPENDITURE ON PLEASURE GOODS
MILLIONS OF 1957 DOLLARS

<u>Quarter</u> <u>Year</u>	I	II	III	IV
1956	385.7998	414.8994	450.9995	577.5996
1957	394.3997	451.4995	494.4998	586.0996
1958	415.2996	471.1995	494.1997	603.7996
1959	431.6995	487.7998	515.1997	619.6997
1960	439.9995	495.5994	527.3997	630.9995
1961	461.9998	533.0996	560.5996	629.9993
1962	485.6997	561.4995	570.4998	658.3997
1963	499.7996	572.1995	595.5996	677.5996
1964	509.4995	578.5999	620.0994	700.7993
1965	533.8997	621.0000	652.4993	754.6995

TABLE II-8
AGGREGATE PRICE INDEX FOR PLEASURE GOODS
(1957=100)

<u>Quarter</u> <u>Year</u>	I	II	III	IV
1956	98.0332	97.9234	97.9901	98.4533
1957	99.0359	99.7586	100.3821	100.8654
1958	102.3521	102.4248	102.7846	103.3475
1959	103.6720	105.5749	107.0311	107.7734
1960	107.9320	107.5214	107.6776	107.8098
1961	107.6042	107.3289	107.9090	108.9583
1962	108.8595	109.0474	109.4069	109.6015
1963	109.2726	109.2396	109.6628	110.4963
1964	110.8805	111.0735	111.6227	113.1990
1965	113.3298	113.6035	114.0560	114.7429

TABLE II-9

PERSONAL DISPOSABLE INCOME (PERSONAL INCOME LESS
PERSONAL DIRECT TAXES) MILLIONS OF DOLLARS

	I	II	III	IV
1956	4330	4755	6017	5051
1957	4801	5133	6013	5327
1958	5184	5592	6397	5741
1959	5528	5972	6760	5976
1960	5812	6035	7025	6212
1961	6071	6292	7023	6625
1962	6459	6874	7896	7014
1963	6737	7272	8498	7511
1964	7400	7571	8732	8022
1965	8084	8214	9749	8943

TABLE II-10
CONSUMER PRICE INDEX OF ALL GOODS AND
SERVICES (1957=100)

	I	II	III	IV
1956	95.10	95.6	96.99	98.5
1957	98.96	100.0	100.5	100.5
1958	101.6	103.0	102.7	103.0
1959	103.8	103.9	104.0	104.5
1960	104.6	105.0	105.1	105.4
1961	105.8	105.8	106.1	105.9
1962	106.4	107.0	107.8	107.6
1963	108.0	108.4	109.6	109.2
1964	109.6	110.1	111.3	110.9
1965	111.4	112.2	113.5	113.4

TABLE II-11
POPULATION OF CANADA
(MILLIONS OF PERSONS)

	I	II	III	IV
1956	15.99	16.08	16.21	16.35
1957	16.48	16.61	16.73	16.85
1958	16.96	17.08	17.18	17.28
1959	17.38	17.48	17.58	17.68
1960	17.77	17.87	17.96	18.05
1961	18.15	18.24	18.32	18.40
1962	18.49	18.57	18.65	18.73
1963	18.81	18.90	18.98	19.07
1964	19.15	19.23	19.32	19.40
1965	19.49	19.57	19.65	19.74

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